Solow Growth Model Project Proposal

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This project will mainly deal with the *Solow Growth Model* in economic. The model gives some insight about capital accumulation and long term economic growth and indicates converges to the the same condition for all countries. This project investigate the numerical solution of the steady state that Canada may get to in the long-run.

The model with some assumption will bring us to the ODE

$$k_t = sAk^{\alpha} - (\delta + n)k \tag{1}$$

which is a first order non-linear ODE system of $k(t)$ with $(s, A, \alpha, \delta, n)$ constant which represents the saving ratio, productivity factor, labour share and capital depreciation rate. The constant terms should be figure out before start to solve for $k(t)$, the capital per captia. We consider the Bayesian approach for estimation of the true parameter which is

$$f_{\Theta|X}(\theta \mid x) = \frac{f_{X,\Theta}(x,\theta)}{f_X(x)} = \frac{f_{X|\Theta}(x \mid \theta) f_{\Theta}(\theta)}{\int f_{X|\Theta}(x \mid \theta') f_{\Theta}(\theta') \, d\theta'}$$

while this is usually not in closed form if the conjugate prior is not chosen. As the parameters are all in a range of $[0, 1]$ (i.e a bunch of ratios), so a Beta distribution should be an ideal candidates. Also we could consider the truncated normal One difficulty here is the computation of the integral of the denominator integral which contains a pdf 4-dimensional random vector. Using quadrature rules here to estimate the expectation of $\theta|X$. May try *trapezoidal, midpoint* and *adaptive* rules for

$$\mathbb{E}(\boldsymbol{\theta} \mid X) = \int f_{\Theta|X}(\boldsymbol{\theta} \mid x) \boldsymbol{\theta} \, d\boldsymbol{\theta}$$

However the rules are not very intuitive for higher dimensional cases. The monte carlo method should be more efficient. Do this for all (s, α, δ, n). For parameter A, since it is not observable, it will be obtained from fitting a linear model from the Cobb-Douglas production function.

The reason for considering of Bayesian's approach is that the model is considering the economy is to goes forward with fixed parameters. When we change some of the parameters and comparing with the status quo, this should be considered as time varying. So it is then reasonable to assign

After the parameters been determined we are able to solve the ODE. The system is non-linear with k^{α} while it is still a ODE, we first get a finite difference scheme by discretization in time

$$D_1 K_j - sAK_j^{\alpha}(\delta + n)K_j = 0$$

and solve the scheme using Newton's method. It's true we can simply put this into a ODE solver while it's interesting to code up the algorithm ourselves. Also we can appeal to the forward Euler's method which is

$$k(t+1) = k(t) + f(t-1, k(t-1))$$

where

$$f(t, k(t)) = sAk(t)^{\alpha} - (\delta + n)k(t)$$

Notice we should consider the stability of the scheme which is neither L or A stable due the the explicit FE method.